Fragmented currents induced by shear flows as heating and dissipation mechanism in flares and the quiet solar atmosphere

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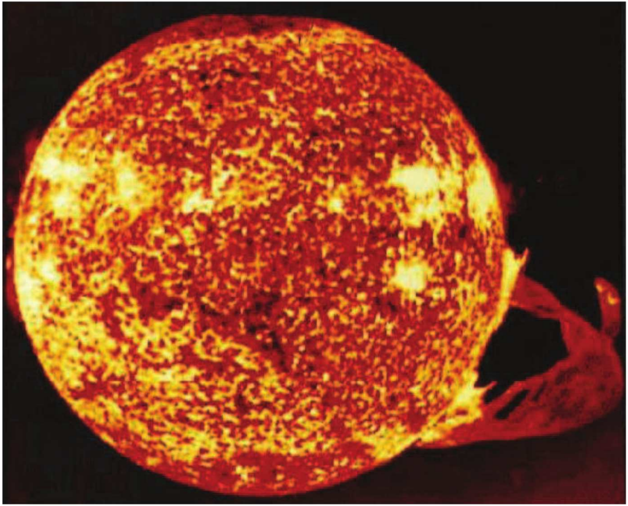
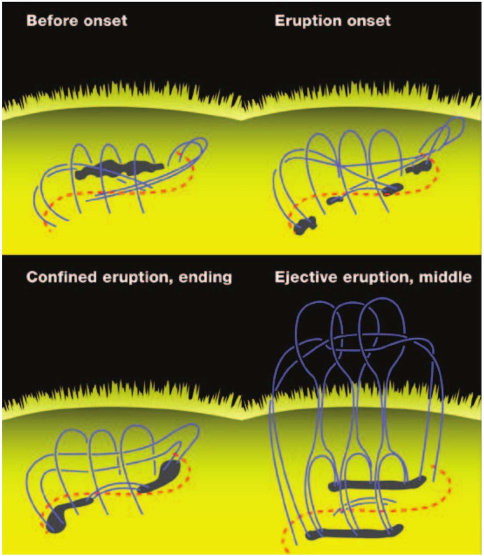
Töravere, April 1, 2015




Before onset
Eruption onset
Confined eruption, ending
Ejective eruption, middle

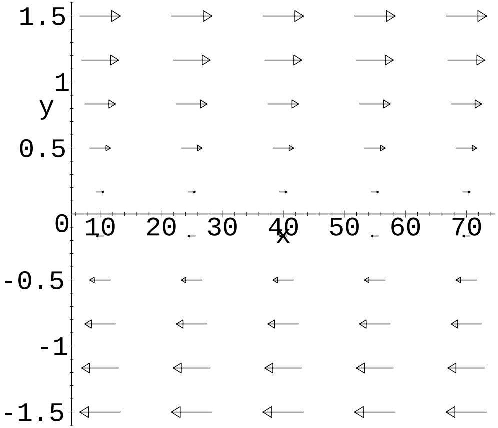
Motivation and remarks

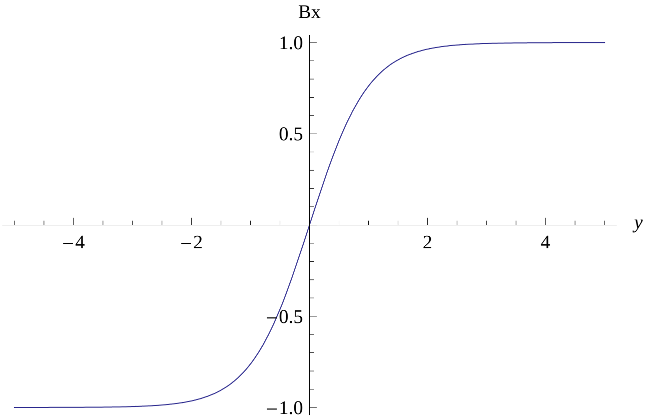
Quasi-static approach is reasonable...

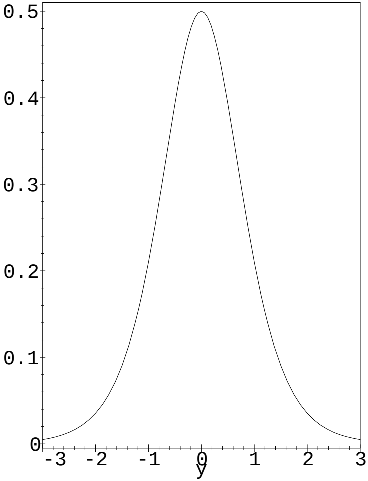
- life time of magnetic structures in plasmas often longer than dynamic time scales, e.g., Alfvénic transit time, see, e.g., Birn & Schindler (1981) or Low (1990)
- flares/CME or magnetotail disruptions (and correlated magnetospheric substorms) take place on comparably short time scale, Birn et al. (1978), Schindler et al. (1983), Platt & Neukirch (1994)
- $\longrightarrow$ *quasi-static approach* is reasonable to understand solar magnetic fields/magnetospheric structures

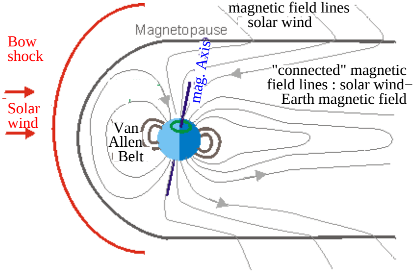
...but quasi-stationary approach include at least also plasma-flows

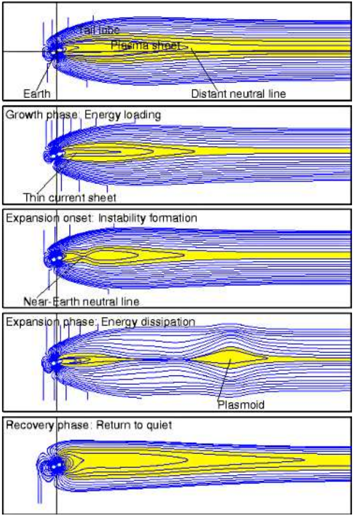
- along loops in the solar corona or along field lines outside the plasma/current sheet of the Earth's magnetotail slow flows can be seen/measured, Baumjohann et al. (1988)
- correlation between static and stationary picture? $\Rightarrow$ yes


1.5
1
y
0.5
0
-0.5
-1
-1.5
10
20
30
40
50
60
70
x


Bx
1.0
0.5
−0.5
−1.0
−4
−2
2
4
y


0.5
0.4
0.3
0.2
0.1
0
-3
-2
-1
0
1
2
3
y

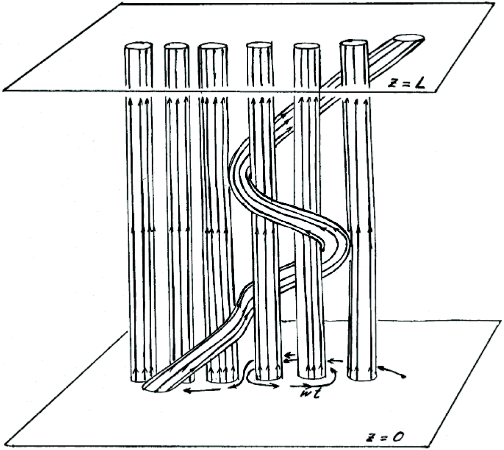

magnetic field lines
solar wind
Magnetopause
Bow
shock
Solar
wind
mag. Axis
"connected" magnetic
field lines : solar wind−
Earth magnetic field
Van
Allen
Belt


Tail lobe
Plasma sheet
Earth
Distant neutral line
Growth phase: Energy loading
Thin current sheet
Expansion onset: Instability formation
Near-Earth neutral line
Expansion phase: Energy dissipation
Plasmoid
Recovery phase: Return to quiet

Motivation for current fragmentation

Solar corona: Heating scenarios (dissipation, acceleration, MR)

- Parker-conjecture vs., e.g. Bogoyavlenskij
- 1D, 2D MHS equilibria $\rightarrow$ singular CS if no symmetry?
- ...or steady state flows?
- CS-heating vs. shocks, compression: not only CS involved in heating (Buechner, 2012)
- but braiding (Wilmot-Smith et al. 2011) or spontaneous stochasticity (Eyink et al. 2011) $\rightarrow$ stirring/diffusion
- in any case, then $\longrightarrow$ dissipation, heating, stochastic MR


z = L
z = 0
ωt

Basic assumptions and equations

Paradigm of ‘large’ and ‘small’ eruptive plasma processes in space:

- 1. Sequence of MagnetoHydro Static (**MHS**) equilibria
- 2. Varying parameter (not necessarily linear in time) up to a value at which a thin or singular current sheet ((T)CS) forms, inducing either
  - a) loss of equilbrium (*Forbes & Isenberg, 1991*)
  - $\Leftarrow$ *Kliem, Lin, Forbes, Priest, Török (2014)* $\Rightarrow$
  - b) instability (*Sturrock, 1989*)
  - c) turbulence (*Lazarian & Vishniac, 1999; Eyink, 2012*)

Examples of eruptive plasma processes in space:

- magnetospheric substorms
- flares and coronal mass ejections
- micro/nanoflares

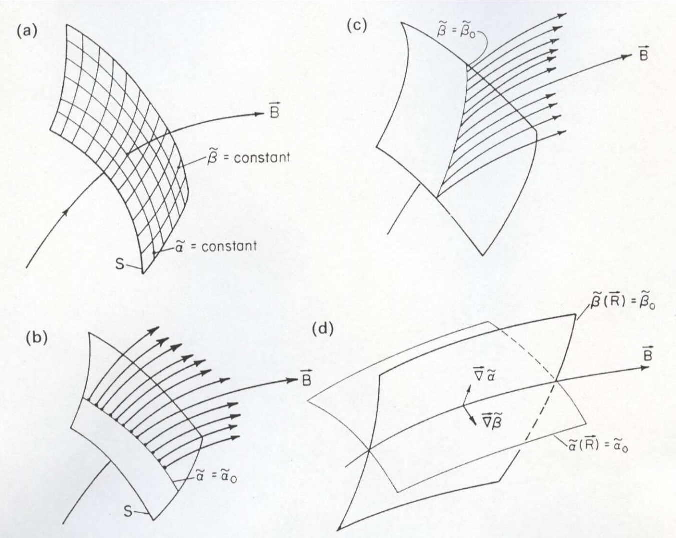
MHS

$$\vec{\nabla} P_{(MH)S} = \vec{j}_S \times \vec{B}_S \quad \Rightarrow \quad \vec{B}_S \cdot \vec{\nabla} P_S = 0$$

Most important properties: Symmetries $\rightarrow$ conservation laws

P_S is conserved along field lines, P_S is so called 'first integral' , i.e.

$$\vec{B}_S = \vec{\nabla}\alpha \times \vec{\nabla}\beta \quad \Rightarrow \quad P_S = P_S(\alpha, \beta)$$


(a)
$\vec{B}$
$\tilde{\beta}$ = constant
$\tilde{\alpha}$ = constant
S
(c)
$\tilde{\beta} = \tilde{\beta}_0$
$\vec{B}$
(b)
$\vec{B}$
$\tilde{\alpha} = \tilde{\alpha}_0$
S
(d)
$\tilde{\beta}(\vec{R}) = \tilde{\beta}_0$
$\vec{B}$
$\vec{\nabla}\tilde{\alpha}$
$\vec{\nabla}\tilde{\beta}$
$\tilde{\alpha}(\vec{R}) = \tilde{\alpha}_0$

MHS without gravitation and flows: Grad-Shafranov equation

Vector potential

With $\vec{B}_S\,(x,y) = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \left(A(x,y)\vec{e}_z\right)$ we find

$$\vec{B}_S = \vec{\nabla} A \times \vec{e}_z \;\Rightarrow\; \vec{B}_S \cdot \vec{\nabla} A = 0$$
$$\Rightarrow\; A(x,y) = \mathrm{const\ are}\,\textit{fieldlines}. \tag{1}$$

Grad–Shafranov equation (*Grad, 1960*, *Shafranov, 1958*)

$$\vec{\nabla} P_S = \vec{j}_S \times \vec{B}_s \;\Rightarrow\; \frac{dP_S}{dA} = -\frac{1}{\mu_0}\Delta A \tag{2}$$

Quasi-static theory – current sheets and field-aligned flows

Current sheet formation with quasi-static model based on the adiabatic MHD-theory developed by Schindler and Birn (1982), extended by Wiegelmann, Schindler, 1995, Becker, Neukirch, Schindler, 2001

- formation of thin current sheets by adiabatic convection

'On the generation of field-aligned plasma flow at the boundary of the plasma sheet' (Schindler and Birn, 1987), (Wiechen and Schindler 1988)

- close to separatrix strong parallel flow occur: (generic aspect) dominant form of plasma transport, the unbalanced perpendicular flow ($\sim$quasi-steady states) requires large parallel flow to establish mass conservation

Quasi-steady current sheet structures with field-aligned flow (Birn, 1992)

- analysis of flows around plamoids

Basic ideal MHD equations

$$\begin{aligned}
\vec{\nabla} \cdot (\rho \vec{v}) &= 0 \,, && (3)\\
\rho \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} &= \vec{j} \times \vec{B} - \vec{\nabla} P \,, && (4)\\
\vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) &= \vec{0} \,, && (5)\\
\vec{\nabla} \times \vec{B} &= \mu_0 \vec{j} \,, && (6)\\
\vec{\nabla} \cdot \vec{B} &= 0 \,, && (7)
\end{aligned}$$

$$\begin{aligned}
\vec{\nabla} \cdot \vec{v} &= 0 \,, && (8)\\
\vec{v} &= \pm |M_A| \vec{v}_A \,, \;\; \vec{v}_A = \frac{\vec{B}}{\sqrt{\mu_0 \rho}} && (9)
\end{aligned}$$

Basic assumptions and conclusions

Basic properties of incompressible and field-aligned flows

$$\vec{\nabla}\cdot\vec{v}=0 \quad \Rightarrow \quad \vec{v}\cdot\vec{\nabla}\rho=0$$

$$\vec{v}=\frac{\pm|M_A|\vec{B}}{\sqrt{\mu_0\rho}} \quad \text{and} \quad \Pi=P+\frac{1}{2}\rho\vec{v}^2 \tag{10}$$

...and therefore some conclusions are...

- the plasma density ρ is constant on field lines
- the Alfvén Mach number M_A is constant on field lines
- the Bernoulli-pressure Π is constant on field lines

Steady-state solutions of field-aligned MHD flows for axissymetric systems and 3D configurations

Transformation method

- Zwingmann 1984 (3D), Euler potential representation reveals Hamiltonian structure
- P.J. Morrison 1986
- Clemente 1993 axissymmetric equilibria with anisotropic pressure
- Throumoulopoulos and Tasso 2000
- Throumoulopoulos and Tasso 1997 – 2005

Self-consistent solutions of the non-linear ideal MHD equations

With the help of known solutions of p_S, B_S, M_A, ρ from the system

$$\vec{\nabla} p_S = \vec{j}_S \times \vec{B}_S, \quad \vec{B}_S \cdot \vec{\nabla}\rho = 0, \quad \text{and } \vec{B}_S \cdot \vec{\nabla} M_A = 0.$$

$$\Downarrow$$

General solution (e.g. Gebhardt & Kiessling, 1992; Nickeler, Goedbloed, & Fahr, 2006; Nickeler & Wiegelmann, 2012)

$$\vec{B} = \frac{\vec{B}_S}{\sqrt{1-M_A^2}}, \qquad p = p_S - \frac{1}{2\mu_0}\frac{M_A^2\left|\vec{B}_S\right|^2}{1-M_A^2}, \tag{11}$$

$$\sqrt{\rho}\vec{v} = \frac{1}{\sqrt{\mu_0}}\frac{M_A\vec{B}_S}{\sqrt{1-M_A^2}}, \qquad \vec{j} = \frac{M_A}{\mu_0}\frac{\vec{\nabla}M_A \times \vec{B}_S}{\left(1-M_A^2\right)^{\frac{3}{2}}} + \frac{\vec{j}_S}{\left(1-M_A^2\right)^{\frac{1}{2}}}. \tag{12}$$

Basic assumptions and conclusions

Basic properties of incompressible and field-aligned flows

$$\vec{\nabla} \cdot \vec{v} = 0 \quad \Rightarrow \quad \vec{v} \cdot \vec{\nabla} \rho = 0$$
$$\vec{v} = \frac{\pm |M_A| \vec{B}}{\sqrt{\mu_0 \rho}} \quad \text{and} \quad \mathit{\Pi} = P + \frac{1}{2} \rho \vec{v}^2 \tag{13}$$

...and therefore some conclusions are...

- the plasma density ρ is constant on field lines
- the Alfvén Mach number M_A is constant on field lines
- the Bernoulli-pressure $\mathit{\Pi}$ is constant on field lines
- the current density changes via the non-canonical mapping! $\rightarrow$ connection to current fragmentation, e.g., Karlický & Bárta (2008); Bárta, Buechner & Karlický (2010), but also by Wiechen, Birk, Lesch (1998); Konz, Wiechen, Lesch (2000); and Poulipoulos, Throumoulopoulos, Tasso (2006)/Throumoulopoulos, Tasso (2010)

Non-canonical transformations (2D) (In 3D Gebhardt & Kiessling 1992)

The definition $\vec{B} = \vec{\nabla}\alpha \times \vec{e}_z$ implies:

The Alfvén Mach number M_A, the mass density ρ and the Bernoulli-pressure $\mathit{\Pi}$ are explicit functions of α

To solve the problem of field-aligned, incompressible flows:

Assuming a substitution/transformation $\alpha = \alpha(A)$ or $A = A(\alpha)$ to transform Euler-equation to the simpler *Grad–Shafranov equation*

..., i.e., leading to (*Nickeler et al., 2006*)

$$\frac{d\mathit{\Pi}}{dA} = -\frac{1}{\mu_0}\Delta A \quad \text{and} \quad \alpha(A) = \pm \int \frac{dA}{\sqrt{1 - M_A^2}} \tag{14}$$

Non-canonical transformations (2D).....

The modified Grad–Shafranov equation

$$\frac{d\Pi}{dA} = -\frac{1}{\mu_0}\Delta A \quad \text{and} \quad \alpha(A) = \pm\int \frac{dA}{\sqrt{1-M_A^2}} \Leftrightarrow \left(\alpha'(A)\right)^2 = \frac{1}{1-M_A^2} \qquad (15)$$

Cooking recipe

- 1. Choose a pressure function Π as a function of A
- 2. Solve Grad–Shafranov equation
- 3. impose a flow on every fieldline $A(x, y) =$const, by choosing a function $M_A(A)$

The effect on the 'new' current distribution

The transformed current

$$j = j_z = -\frac{M_A M_A' \left(\vec{\nabla} A\right)^2}{\left(1 - M_A^2\right)^{3/2}} - \frac{\Delta A}{\left(1 - M_A^2\right)^{1/2}} \tag{16}$$

$$= -\Delta\alpha = -\alpha'' \left(\vec{\nabla} A\right)^2 - \alpha' \Delta A. \tag{17}$$

$$M_A(A) = \sum_k m_k(A) + \int m(A)\, dA\ , \tag{18}$$

$$M_A^2 = 1 - \frac{1}{\left(\frac{d\alpha}{dA}\right)^2} \quad \Leftrightarrow \quad \left(\alpha'(A)\right)^2 = \frac{1}{1 - M_A^2} \tag{19}$$

Example: Nonlinear MHS-equilibrium with two different fluxropes, bipolar bottom region and Harris-sheet behaviour for $x \to \infty$

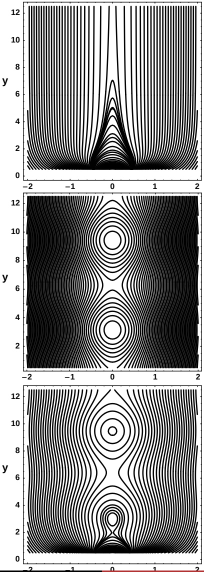
A solution of

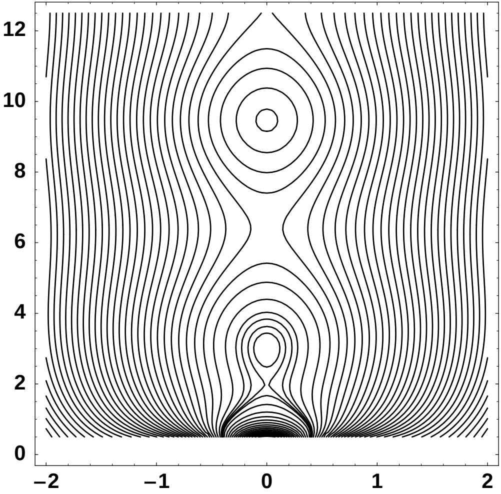
$$\Delta A = \exp\left(-2A\right) \tag{20}$$

with the above mentioned properties is (Nickeler et al. 2013)

$$A = \ln \frac{\sqrt{1+\delta^2}\cosh\left[x\left(1+\frac{d}{r^2}\right)\right] + \delta\cos\left[y\left(1-\frac{d}{r^2}\right)\right]}{\sqrt{\frac{(d^2-2d(x^2-y^2)+r^4)}{r^4}}} \; , \tag{21}$$

where $r^2 = x^2 + y^2$.

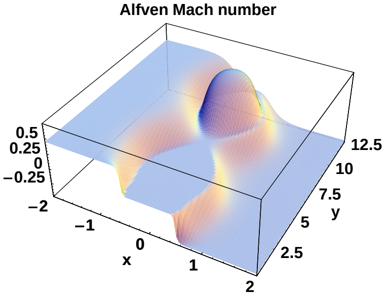

y
0
2
4
6
8
10
12
−2
−1
0
1
2

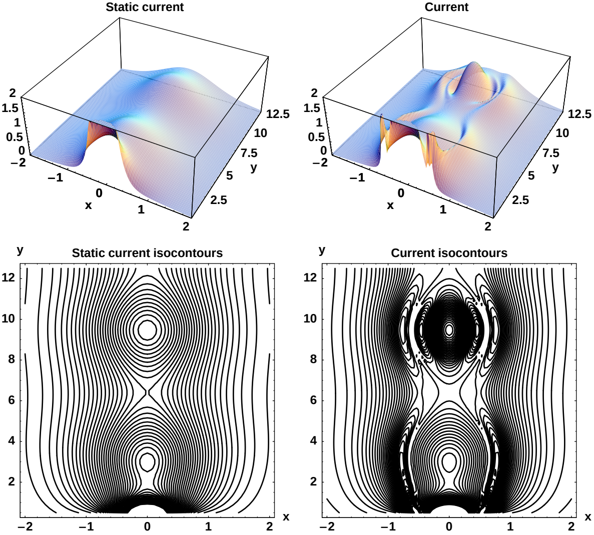

12
10
8
6
4
2
0
–2
–1
0
1
2

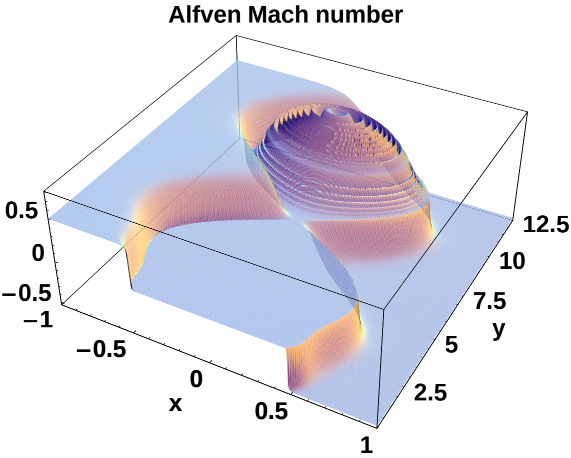
$A_{\rm sep} = 0.0875; A_{\rm b} = 0.1; f = 1 - 0.1\sin(1.1A); f_{\rm p} = 1$

Specifications of Mach number profile covering the region in x and y as defined by the MHS configuration (see Figure)

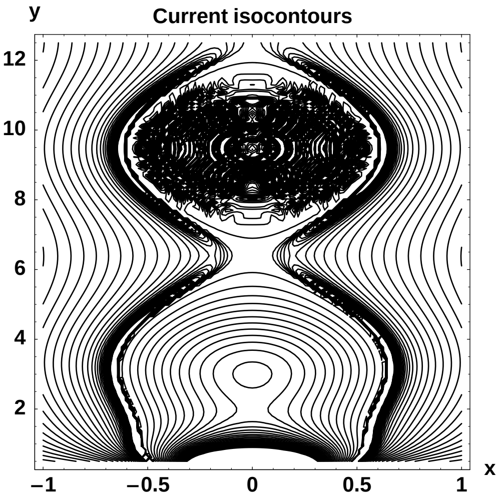
$$M_A = \begin{cases} -0.5f\left(1 - \dfrac{1}{1+\left(\dfrac{A-A_{\rm sep}}{A_b}\right)^2}\right) & \text{for} \quad A > A_{\rm sep}\,, \quad x > 0 \\ 0.5f\left(1 - \dfrac{1}{1+\left(\dfrac{A-A_{\rm sep}}{A_b}\right)^2}\right) & \text{for} \quad A > A_{\rm sep}\,, \quad x < 0 \\ 0.9f_p\left(1 - \dfrac{1}{1+\left(\dfrac{A-A_{\rm sep}}{A_b}\right)^2}\right) & \text{for} \quad A < A_{\rm sep}\,, \quad y > 6.5 \\ 0 & \quad\quad\quad \text{elsewhere} \end{cases}$$

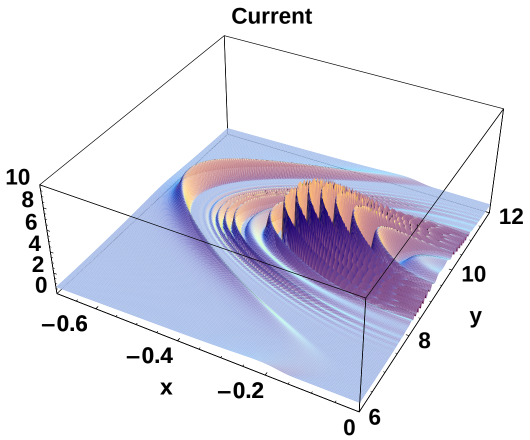


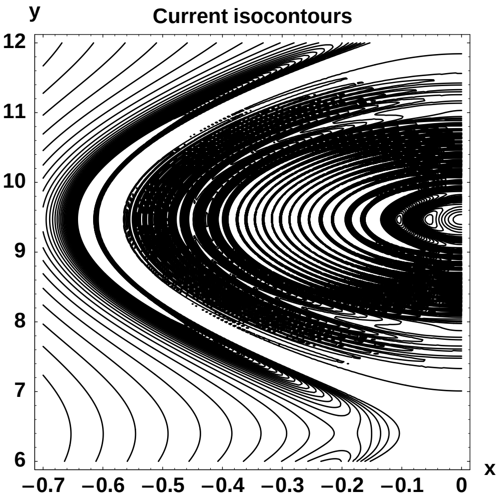

Static current
Current
2
1.5
1
0.5
0
−2
−1
0
1
2
x
2.5
5
7.5
10
12.5
y
Static current isocontours
Current isocontours
y
12
10
8
6
4
2
x
−2
−1
0
1
2

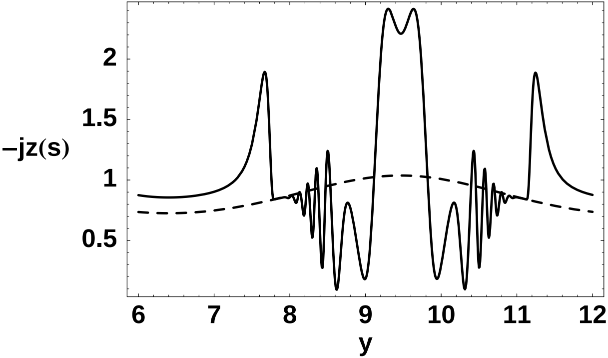

Alfven Mach number
0.5
0
−0.5
−1
−0.5
0
x
0.5
1
2.5
5
y
7.5
10
12.5

$A_{\mathrm{sep}} = 0.0875; A_{\mathrm{b}} = 0.01; f = 1; f_{\mathrm{p}} = 1 - 0.1\sin(1/(A^2 + 0.01))$




Current
10
8
6
4
2
0
−0.6
−0.4
x
−0.2
0
6
8
y
10
12


Current isocontours
y
12
11
10
9
8
7
6
x
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
0


2
1.5
−jz(s)
1
0.5
6
7
8
9
10
11
12
y

The inverse problem & solution: (almost) arbitrary current profiles

Assuming

$$\lim_{x,y\to\infty} \vec{B}_S = \vec{B}_{S\infty}$$

we define

$$|\vec{B}_{S\infty}| = B_{S\infty}(A).$$

The Limes has to be smooth. Then we can introduce the asymptotical current via

$$\vec{\nabla} \times (\lim_{x,y\to\infty} \vec{B}_S) = \lim_{x,y\to\infty}(\vec{\nabla} \times \vec{B}_S) = \lim_{x,y\to\infty} \vec{j}_S,$$

with

$$\lim_{x,y\to\infty} |\vec{j}_S| = j_{S\infty}(A) = {P_S}'(A).$$

Solution of the inverse problem

Asymptotical 1D limit for 2D (or 3D) magnetic field structures

$$-j(A,y) = \alpha''(A)B_S^2(A,y) - \alpha'(A)j_S(A) \tag{22}$$

$$\downarrow \lim x, y \to \infty$$

$$-j_\infty(A) = \alpha''(A)B_{S\infty}^2(A) - \alpha'(A)j_{S\infty}(A)\,. \tag{23}$$

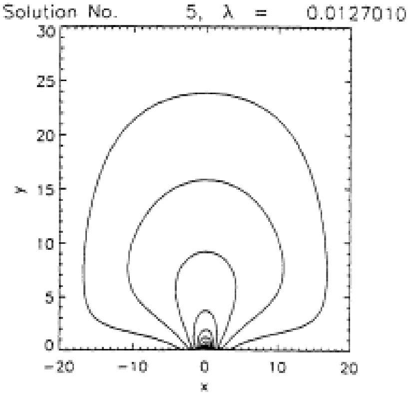
with the solution (Nickeler et al. 2013):

$$\alpha'(A) = \frac{\int \exp\left(\int -\frac{j_{S\infty}}{B_{S\infty}^2}\,dA\right)\left(-\frac{j_\infty}{B_{S\infty}^2}\right)dA + C_0}{\exp\left(\int -\frac{j_{S\infty}}{B_{S\infty}^2}dA\right)}\,. \tag{24}$$

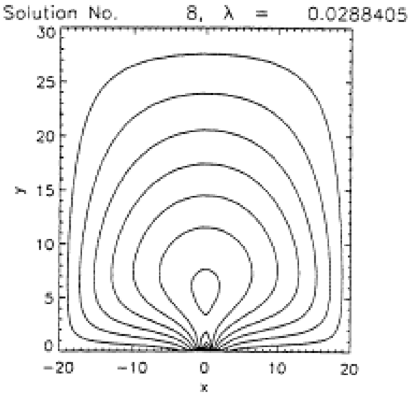
For every arbitrary current distribution j_∞ with $|\alpha'| \geq 1$ a steady-state flow exists, which enables Ohmic heating and dissipation. Suitable parameterizations of $j_\infty(A)$ are available!

Summary and conclusions

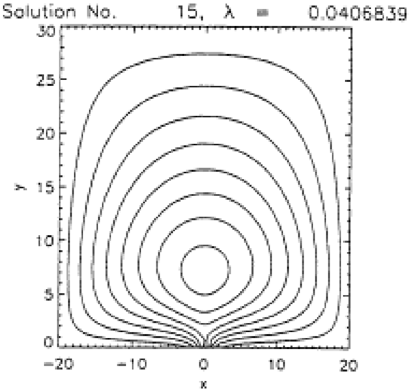
- MHD equilibria are connected with conserved quantities and Hamiltonian structures, including symmetries
- from MHS we can via mapping technique calculate steady-state ideal MHD flows
- via (shear) flows and (therefore) strong vortex sheets strong current sheets can be generated
- general proof that these current sheets can and will be highly fragmented
- These current sheets are locations for efficient Ohmic heating and dissipation
- Further work
  - finding ‘optimal’ current distribution for converting the energy via Ohmic heating
  - Generalised Ansatz for finding transformations for 3D equilibria
  - Transformation from 2D to 3D and corresponding fragmentation?
  - 2D solutions for *compressible* or non-field-aligned flows with fragmentation?
  - exact time-dependent non-linear solutions?



(Romeou & Neukirch, 1999)



(Romeou & Neukirch, 1999)



(Romeou & Neukirch, 1999)

Examples

Potential fields: Laplace equation

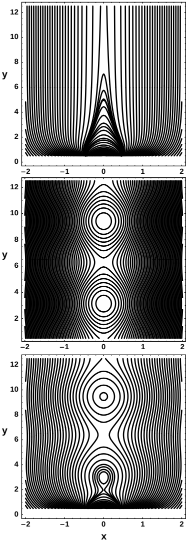
$$\Delta A = 0 \tag{25}$$

Fields with linear current: Helmholtz equation

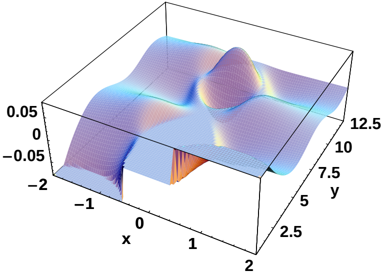
$$\Delta A = c^2 A \tag{26}$$

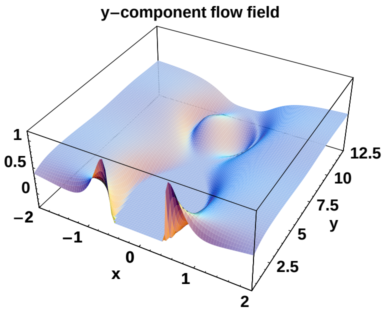
Field with non-linear current: (often called ‘Liouville’s equation’), Liouville 1853

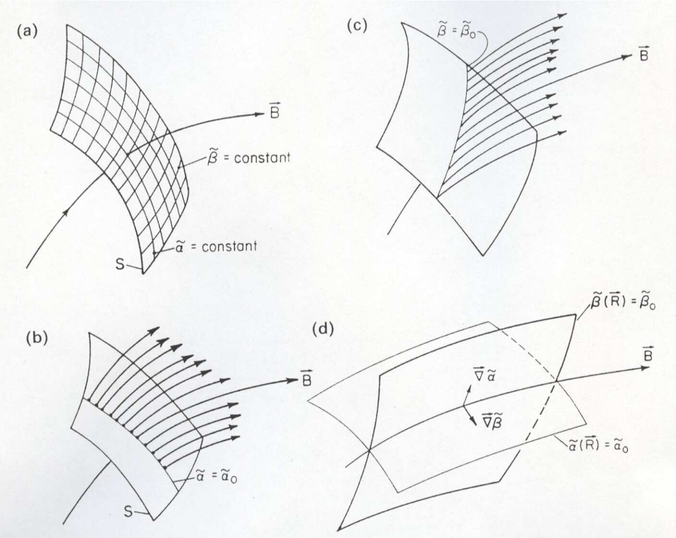
$$\Delta A = \exp\left(-2A\right) \tag{27}$$


y
x
12
10
8
6
4
2
0
−2
−1
0
1
2

x–component flow field




y−component flow field
1
0.5
0
−2
−1
0
x
1
2
2.5
5
7.5
y
10
12.5


(a)
B
β̃ = constant
α̃ = constant
S
(c)
β̃ = β̃₀
B
(b)
B
α̃ = α̃₀
S
(d)
β̃(R) = β̃₀
B
∇α̃
∇β̃
α̃(R) = α̃₀

Magnetohydrostatics (MHS) in 3D and conserved quantities: the Euler-or Clebsch potentials

MHS equations with Hamiltonian structure

$$\vec{\nabla}\beta \cdot \vec{\nabla} \times (\vec{\nabla}\alpha \times \vec{\nabla}\beta) = \mu_0 \frac{\partial P_S}{\partial \alpha}$$

$$\vec{\nabla}\alpha \cdot \vec{\nabla} \times (\vec{\nabla}\alpha \times \vec{\nabla}\beta) = -\mu_0 \frac{\partial P_S}{\partial \beta} \tag{28}$$

Hamiltonian structure, conserved values, potential representation similar to GS-theory

Steady MHD equilibria with flow in 2D: Equilibrium equations similar to Schindler, Birn, Janicke 1983 flow or Tsinganos 1980–1984

Representation of fields

Representation of magnetic fields $\vec{B} = \vec{\nabla}\alpha \times \vec{e}_z + B_z\vec{e}_z$, velocity field $\sqrt{\rho}\vec{v} = \vec{w} = \vec{\nabla}\zeta \times \vec{e}_z + w_z\vec{e}_z$, generalized pressure $\Pi = p + \rho/2\,(v_x^2 + v_y^2) + B_z^2/(2\mu_0)$

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Equilibrium equations: $\vec{\nabla}\Pi = -\frac{1}{\mu_0}\Delta\alpha\,\vec{\nabla}\alpha + \Delta\zeta\,\vec{\nabla}\zeta$ (momentum equation), $-\vec{\nabla}\phi_e(x,y) + E_z\vec{e}_z + \vec{w}\times\vec{B}/\sqrt{\rho} = \vec{0}$ (Ohm's law)

$$\begin{aligned}\frac{\partial \Pi}{\partial \alpha} &= -\frac{1}{\mu_0}\Delta\alpha + \frac{\partial\zeta}{\partial\alpha}\Delta\zeta \quad \text{and} \quad \frac{\partial \Pi}{\partial \phi} = -\rho + \frac{\partial\zeta}{\partial\phi}\Delta\zeta \\ \frac{\partial(\zeta,\alpha)}{\partial(x,y)} &= \sqrt{\rho}E_z \quad + \text{ constraints for the z-components}\end{aligned} \qquad (29)$$

Basic assumptions and equations

conditions for incompressible and field-aligned flows

$$\vec{\nabla} \cdot \vec{v} = 0$$

$$v = \frac{\pm|M_A|\vec{B}}{\left(\sqrt{\mu_0 \rho}\right)} \tag{30}$$

$$\Pi = P + \frac{1}{2}\rho\vec{v}^2 \tag{31}$$

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...and the remaining equations

$$\vec{B} \cdot \vec{\nabla} M_A = 0\,, \tag{32}$$

$$\vec{\nabla}\Pi = \frac{\left(1 - M_A^2\right)\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}}{\mu_0} - \frac{|\vec{B}|^2}{2\mu_0}\vec{\nabla}\left(1 - M_A^2\right)\,, \tag{33}$$

$$\vec{\nabla} \cdot \vec{B} = 0\,, \tag{34}$$

Non-canonical transformations (2D)

Defining:

$\vec{B} = \vec{\nabla}\alpha \times \vec{e}_z$ and assume $\alpha = \alpha(A)$ or $A = A(\alpha)$

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To solve:

$$\vec{\nabla}\Pi = -\frac{\left(1-M_A^2\right)\Delta\alpha\vec{\nabla}\alpha}{\mu_0} - \frac{|\vec{\nabla}\alpha|^2}{2\mu_0}\vec{\nabla}\left(1-M_A^2\right) \tag{35}$$

$$= -\frac{\left(1-M_A^2\right)\left(\alpha''\left(\vec{\nabla}A\right)^2 + \alpha'\,\Delta A\right)\alpha'\vec{\nabla}A}{\mu_0} - \frac{\alpha'^2|\vec{\nabla}A|^2}{2\mu_0}\vec{\nabla}\left(1-M_A^2\right)$$

$$= -\frac{\left(1-M_A^2\right)}{\mu_0}\alpha'^2\Delta A\vec{\nabla}A - \frac{\left(\vec{\nabla}A\right)^2\vec{\nabla}A}{2\mu_0}\left[\left(1-M_A^2\right)\alpha'^2\right]' . \tag{36}$$